planes are brought closer together. The coupling factor, as shown in Fig. 7, is determined from:

$$C_{\mathrm{dB}} = -20\log_{10}\left[\frac{\dfrac{K(k_3')}{K(k_3)} - \dfrac{K(k_4')}{K(k_4)}}{\dfrac{K(k_3')}{K(k_3)} + \dfrac{K(k_4')}{K(k_4)}}\right] \quad (11)$$

The coupling is shown to increase as the shielding ground planes are moved away from the coupled lines.

III. Conclusion

Two new monolithic multilayer coupling structures have been presented and their design characteristics have been derived using direct analytical formulas. These closed form expressions have been used to investigate the variations in structure mode impedances and coupling coefficients. The placing of coupled lines, perpendicular to their ground plane(s), provides an improved alternative to coplanar edged coupled lines, where conductor edge current crowding needs to be minimized.

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Rigorous Analysis of Iris Coupling Problem in Waveguide

Rong Yang and A. S. Omar

***Abstract*—In this short paper we present a new class of simple basis functions which explicitly take the edge conditions into consideration for solving waveguide iris coupling problem by using moment method. The good agreement between the results for some special cases for both parallel-plate waveguide and rectangular waveguide from the present work and that from previous publications demonstrate the correctness of the choice of the basis functions. Compared with previously published basis functions the basis functions introduced here are characterized by their simple form, generality and still with a similar fast convergence behavior.**

I. Introduction

Waveguide iris coupling mechanism is frequently employed in building microwave components such as waveguide filters and impedance matching systems. Various analysis approaches like Conformal Mapping, Variational Technique, Singular Integration Equation Method, Mode Matching Method and Moment Method have been developed in the past decades with success in dealing with such problems.

The core of the moment method for solving waveguide iris coupling problem lies in a suitable choice of a set of basis functions to represent the tangential electric field behavior in the plane of the coupling iris. A proper choice of such basis functions can drastically reduce the computation efforts with a faster convergence of the results. It has been shown in [1] and [2] that a set of basis functions which account for the edge condition of the coupling aperture can be of such an effect. But their choices of basis functions are complicated and not straightforward. In this paper we present a new class of simple-form basis functions which explicitly take the edge conditions of the coupling iris into consideration. Compared with the previous choices of basis functions the present one is free from complexity and more universal but with similar fast convergence behavior.

II. General Theory and choice of Basis Function

The moment method is a successful one in solving iris coupling problem and a detail description of the method can be found in [4]. Only a very brief introduction of the method will be made in this short paper for the sake of brevity.

The transverse electromagnetic field in the waveguides at both sides of the coupling iris are expanded in terms of the corresponding waveguide eigen modes. The electric field of the coupling aperture is expanded with respect to a set of suitable basis functions. The equality of the tangential electric fields at the two sides of the iris to the aperture field as well as the continuity of the tangential magnetic fields across the iris along with the application of Galerkin's method leads to an infinite set of algebraic equations relating the incident and reflected modal amplitudes in both waveguides, from which a description of the coupling structure in terms of e.g. the S parameter can be derived. A truncation of the above infinite system of equation must be made before a numerical evaluation is to be carried out.

Assuming that the coupling iris is of zero thickness the basis functions which are supposed to account for the singular behavior

Manuscript received October 22, 1991; revised June 9, 1992.
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IEEE Log Number 9204496.

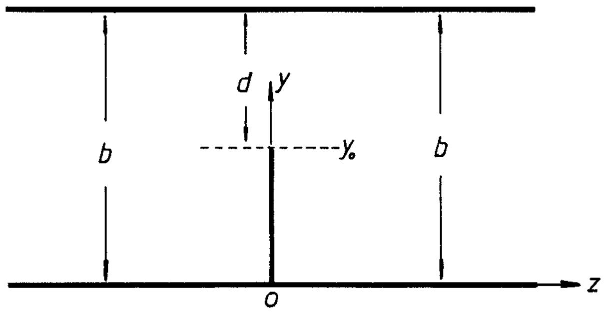

Fig. 1. Infinitively extended parallel-plate with a capacitive semi-diaphragm.

at the edge must satisfy the 0° corner edge conditions. If $x = x_0$ denotes an edge, the basis function denoting the aperture electric field component normal to the edge can be chosen to be of the form

$$\frac{\cos[j\pi(x - x_0)/d]}{\sqrt{(x - x_0)}}, \qquad (j = 0, 1, 2, \cdots)$$

while those for the aperture electric field component parallel to the edge are

$$\frac{\sin[j\pi(x - x_0)/d]}{\sqrt{(x - x_0)}}, \qquad (j = 1, 2, 3, \cdots)$$

which guarantee the satisfaction of the field edge properties.

III. Numerical Results

For the numerical evaluation we choose the waveguide at both sides of the iris to be rectangular with the same dimensions and without offset. Both waveguides are assumed to be lossless.

We consider first the structure of a parallel-plate waveguide of infinite extent with a capacitive semi-diaphragm as shown in Fig. 1. A TEM-mode of unit amplitude is assumed to be incident from the left-hand side of the waveguide. Since the discontinuity of the coupling iris is confined in the y-direction, the excited higher order modes in the iris plane due to an x-independent incident wave are also x-independent. The following basis functions can be chosen to represent the aperture transverse electrical field:

$$e_y = \frac{\cos[i\pi(y - y_0)/d]}{\sqrt{(y - y_0)(y + y_0)}}, \qquad (i = 0, 1, 2, \cdots)$$

where $y = y_0$ is the iris edge. It can be demonstrated that an introduction of an image edge at $y = -y_0$ will not change the characteristics of the coupling structure while it can remarkably reduce the numerical computation efforts.

Table I shows the calculated normalized susceptance as a function of the number of basis functions for such a capacitive semi-diaphragm in parallel-plate waveguide together with the results from [2] and [3]. The present numerical results compare very well to the exact solution in [5] with $B = j$ 1.5931. It is noted that the convergence effect of the here introduced basis functions is similar to that of [2] and [3] and only two to three of such basis functions are sufficient to produce an accurate result. However the convergence pattern of the present basis functions is different from that in the literature mentioned above. Both of the results in [2] and [3] approach the exact value from above while the present one approaches it from below. This can be clearly seen in the table.

Next a symmetrical capacitive obstacle in a rectangular waveguide is considered as shown in Fig. 2. For this structure the metal strip in the waveguide forms two coupling apertures separated by the strip. According to the geometry of the structure, when a TE_{10} dominant

TABLE I
Susceptance of a Capacitance Semi -Diaphragm as a Function of the Number of Basis Functions $\cdots$, Parameter: $\lambda = 2.5b, d = 0.5b$

j	0	1	2	3	4
present result	1.5844	1.5930	1.5931	1.5931	1.5931
Leong *et al* [2]	1.5935	1.5934	1.5931	1.5931	1.5931
Lyapin *et al* [3]	1.6123	1.5932	1.5931	1.5931	1.5931

mode is incident from the left side only TE_{1n} and TM_{1n} modes can be excited at the iris plane. For the lower aperture we choose the basis function of the form

$$e_x = \cos(\pi x/a) \cdot \frac{\sin[j\pi(y_1 - y)/d]}{\sqrt{(y_1 - y)(y_1 + y)}}, \qquad (j = 1, 2, 3, \cdots)$$

$$e_y = \sin(\pi x/a) \cdot \frac{\cos[j\pi(y_1 - y)/d]}{\sqrt{(y_1 - y)(y_1 + y)}}, \qquad (j - 0, 1, 2, \cdots)$$

while for the upper aperture:

$$e_x = \cos(\pi x/a) \cdot \frac{\sin[j\pi(y - y_2)/d]}{\sqrt{(y - y_2)(y + y_2)}}, \qquad (j = 1, 2, 3, \cdots)$$

$$e_y = \sin(\pi x/a) \cdot \frac{\cos[j\pi(y - y_2)/d]}{\sqrt{(y - y_2)(y + y_2)}}, \qquad (j = 0, 1, 2, \cdots)$$

$j = 0, 1, 2, 3, \cdots$

The normalized shunt susceptance of a symmetrical capacitive strip in a rectangular waveguide as a function of the strip width is shown in Fig. 2. Three different curves for different values b/λ_g are plotted. The numerical results obtained here agree well with the results from *Marcuvitz's Waveguide Handbook* [1] within the plotting accuracy. A total number of 40 modes (both TE and TM) are used in the calculation. Similar to that for the case of parallel-plate waveguide two or three of the above basis functions for each aperture are enough to yield a quite accurate result.

Due to their simple form the suggested basis functions are very suitable to deal with more complicated coupling structures such as resonant iris with rectangular aperture(s). A metal diaphragm with a rectangular opening in a rectangular waveguide exhibits a susceptance-frequency characteristics similar to that of a parallel resonant circuit shunting the waveguide. For a centered rectangular resonant window in a rectangular waveguide the resonant frequency can be calculated with the empirical equation

$$\frac{a}{b}\sqrt{1 - \left(\frac{\lambda}{1.97a}\right)^2} = \frac{a_1}{b_1}\sqrt{1 - \left(\frac{\lambda}{1.97a_1}\right)^2} \qquad (1)$$

The definition of the different parameters are shown in Fig. 3. For a dominant mode incidence the higher order modes excited in the iris plane are both TE and TM of any order due to the inhomogeneity of the junction in both x- and y-directions. To account for the aperture edge properties the following basis functions are chosen to expand the transverse electric field at the iris plane

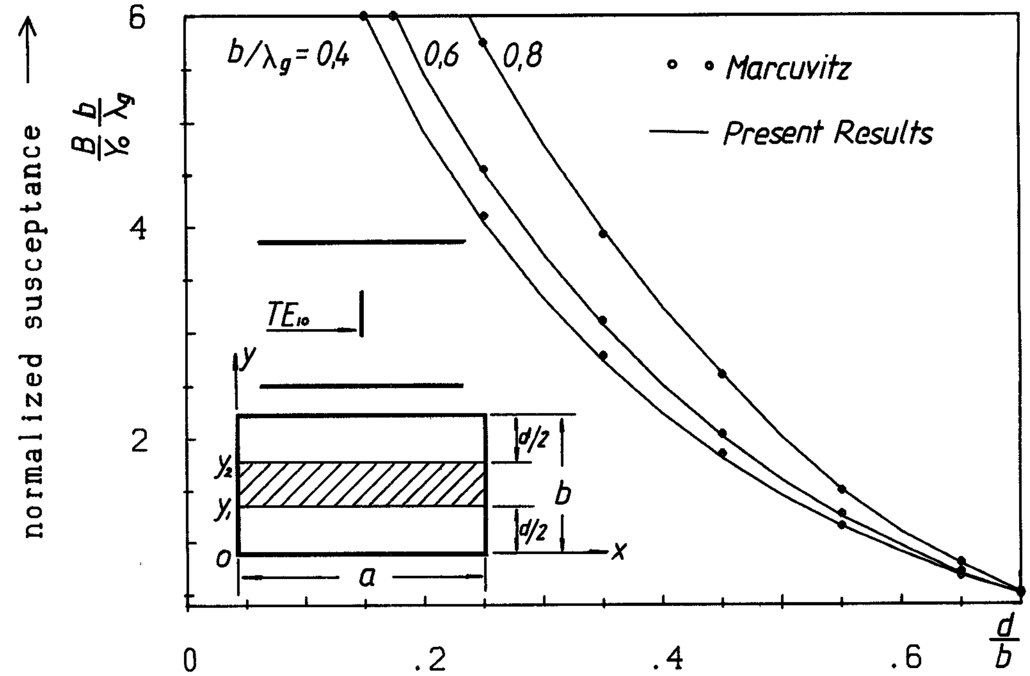

Fig. 2. Normalized shunt susceptance of a symmetrical capacitive obstacle in a rectangular waveguide.

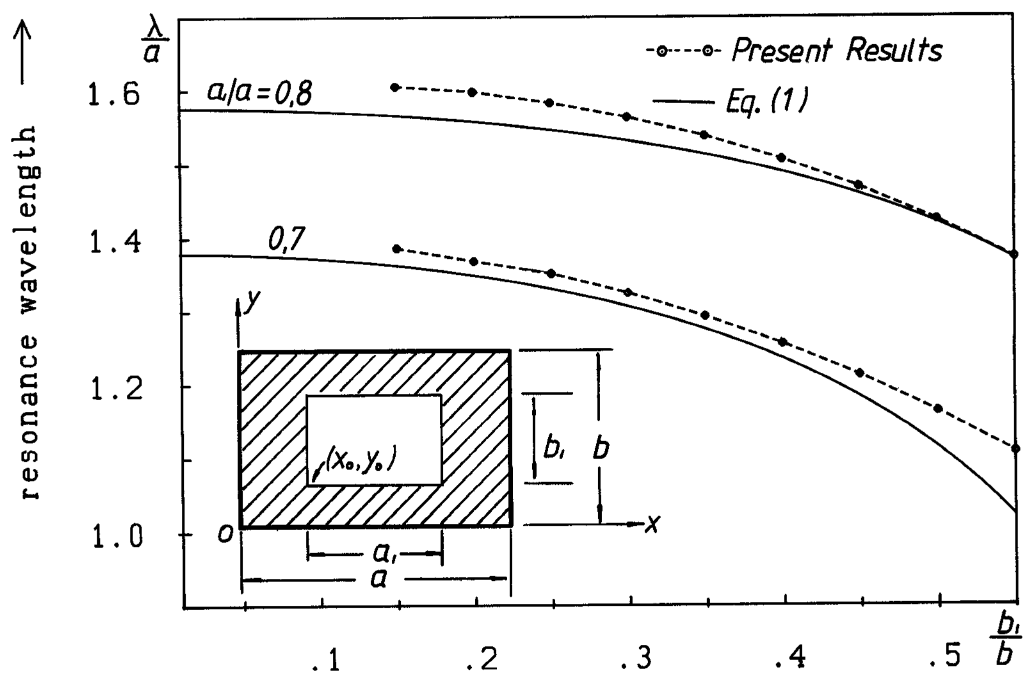

Fig. 3. Resonance wavelength of centered rectangular window in rectangular window in rectangular waveguide as a function of window dimension.

$$e_x = \frac{\cos[i\pi(x-x_0)/a_1]}{\sqrt{(x-x_0)[1-(x-x_0)/a_1]}} \cdot \frac{\sin[j\pi(y-y_0)/b_1]}{\sqrt{(y-y_0)[1-(y-y_0)/b_1]}}$$
$$(i = 0, 1, 2, \cdots;\ j = 1, 2, 3, \cdots)$$
$$e_y = \frac{\sin[i\pi(x-x_0)/a_1]}{\sqrt{(x-x_0)[1-(x-x_0)/a_1]}} \cdot \frac{\cos[j\pi(y-y_0)/b_1]}{\sqrt{(y-y_0)[1-(y-y_0)/b_1]}}$$
$$(i = 1, 2, 3, \cdots;\ j = 0, 1, 2, \cdots)$$

The coordinates (x_0, y_0) correspond to the lower left corner of the aperture.

Fig. 3 gives the calculated resonant frequency of a rectangular resonant window in a rectangular waveguide as a function of the resonant window dimension together with the resonant frequency calculated from (1). It can be noted that the results from the two approaches coincides well with each other with a difference of about one to two percent. Since a discrepancy of a few percent are expected between the result from (1) and that of experiment [6], the results obtained by the present numerical method can be considered to be of good accuracy. Equation (1) is an empirical one which considers only the effect of the dominant mode and it is not likely to give good results when the operating frequency approaches the cutoff frequency of the next higher order mode. This phenomenon can also be observed in Fig. 3. For the curve $a_1/a = 0.7$ when λ/a approaches unity (the cutoff wavelength of the TE_{20} mode) the difference between the two curves becomes bigger. Here again, only two or three basis functions are included in the computation.

In the course of the numerical calculation the most time consuming procedure is the matrix inversion. The size of the matrix to be inverted is determined by the number of the used basis function. Two or three basis functions imply that the matrix to be inverted is of the order 2×2 or 3×3. The numerical analysis with the suggested basis functions is consequently very computationally time saving. The typical CPU time required for one point calculation with as much as 1600 total waveguide modes is about 20 seconds on a CONVEX computer.

IV. Conclusion

A new class of simple-form basis functions are presented for solving waveguide iris coupling problem by using the moment method. A study for different coupling structures using these basis functions shows good agreement with the already published results. Compared with the basis functions published before the basis functions introduced here are characterised by their simple forms, generality and fast convergence which are consequently suitable for analysing relatively complex coupling structures.

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Numerical Electromagnetic Inverse-Scattering Solutions for Two-Dimensional Infinite Dielectric Cylinders Buried in a Lossy Half-Space

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***Abstract*—An approach to microwave imaging in a half-space geometry and for infinite dielectric cylinders buried in a lossy medium is proposed. The two-dimensional integral-equation for the inverse-scattering problem is discretized by the moment method. The resulting ill-conditioned system is solved by pseudoinversion. A multi-incidence process based on the invariance of the Green matrix to the incident field is described. Results of some numerical simulations, assuming a noisy environments, are reported and discussed.**

I. Introduction

In this paper, a two-dimensional integral formulation of inverse scattering in a half-space is proposed for microwave imaging of buried objects. This technique is of great interest in many geophysical and civil-engineering fields. Detection of cables and pipes (plastic materials) is a significant example. In the past, some works dealt with the problem of identifying nonmetallic structures by radar [1], [2], and an interesting method for microwave imaging of buried objects was proposed in [3]. Moreover, a diffraction tomography methodology previously developed for medical applications, was proposed for electromagnetic imaging of buried cylindrical inhomogeneities [4].

In the present paper, the theory of inverse scattering is applied to solve the integral equation, whose unknown terms are the products of the object functions by the total electric field. After considering an equivalent current density to model the investigation domain, the resulting integral equations are solved numerically by the moment method (MoM) [5].

In the last few years, several moment-method-based inverse-scattering solutions for many different situations have been presented. Ghodgaonkar *et al.* [6] developed a method for imaging 3-D biological targets; Ney *et al.* [7] used the pseudoinversion transformation to retrieve the polarization current in mono- and two-dimensional scatterers. Moreover, two works by Guo and Guo [8], [9] furnished a theoretical background to develop reconstruction algorithms. Finally, the authors of the present paper proposed an approach to the reconstruction of unknown dielectric scatterers in free space [10]. In this work, the possibility of determining the dielectric properties of unknown objects buried in a half-space is explored. Input data are obtained by measuring the values of the scattered electric field inside an observation domain located near the boundary between the different media. A TM-wave incident electric field is used to illuminate the unknown objects. A multi-illumination-angle imaging process is proposed. Since the Green matrix is invariant to the incident electric field vector, this multi-illumination process does not require an increase in computational resources. Another interesting feature lies in the possibility of computing off line (and once for all) the pseudoinverse matrix, after fixing the investigation and observation domains.

Manuscript received March 2, 1992; revised May 28, 1992.

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IEEE Log Number 9204497.